vice to Brussels Airport and the Tokyo monorail to Haneda Airport is only comparable to highway travel. The new installations to Cleveland Hopkins and London-Heathrow are expected to save considerable travel time compared to highway travel, according to advance estimates.

In this survey, airline passengers account for 27 per cent of all the airport trips, employees 36 per cent, and all others (including visitors) 37 per cent. In general, the planned new rail links will be of service to the airline passengers and "all others" if they are exclusively oriented to central business district-to-airport travel. If links are added to serve intermediate districts, more employee traffic would be accommodated, but en route stops would delay travel and thus inhibit the full development of airline passenger potential. Estimates vary widely with local circumstances and conditions, but as reported, rail service handles anywhere from 20 to 80 per cent of total traffic between central business districts and airports.

The results of this survey suggest that rail access can provide valuable service connections in some cases, but only the most careful analysis of an individual situation can determine its potential patronage and feasibility. However, it is clear that highway travel will continue to be important from the central business district because of personal preferences for automobiles and taxi services. In addition, highways will, of course, continue to be indispensable for access from all non-CBD areas served by the airport.

Perhaps the most exhaustive study of airport access of recent years has been carried out by the British Airports Authority with respect to London-Hearthrow and London-Gatwick airports. These reports, which tabulate very large samples of passengers, visitor/spectators and employees, provide a very comprehensive data base and are particularly significant in emphasizing the very heavy load imposed on the access systems by employees. Whereas, for Heathrow in 1965, daily trips by air passengers in each direction totalled 20,500 in summer and 9200 in winter and visitor/spectators varied from 7100 to 3200, the employees represented 35,000 trips all year round. It is anticipated by the BAA that airline passenger and visitor/spectator traffic will increase by two and a half times by 1975, while the employee population will increase to 47,000, again on a year-round basis. The important point for Heathrow, or for any other major airport, is that while an efficient rail link may attract substantial numbers of passengers and visitors/spectators away from automobiles and taxis, the employee population, in the main, does *not* commute from the CBD, and the likelihood of attracting the employees from private automobiles to any rail access system, even at the expense of downgrading the service by introducing intermediate stops, seems very poor. It may thus be that airport planners will continue to face mounting problems of access and parking created by employee vehicles without being able to siphon this traffic off to any other mode. It should be pointed out that, generally, employee traffic peaks substantially over-lap passenger and visitor/spectator peaks.

It should be noted that of the airports surveyed, 15 are outside the United States. Of the seven major American airports responding, only Cleveland has an existing direct rail link to the terminal complex. The other systems, like Boston's Logan Airport, involve a subway-bus routing or, like San Francisco, are in proposal or study stages. This simply means that the bulk of major airports in the United States will continue to be reached by highway systems and, in an overwhelming percentage, by automobile and taxi. The following comments by Mr. Peavey of Port of New York Authority with respect to the decline in airport bus passengers are significant.

1. Travel to JFKIA

In 1956, Carey handled 63% of the air passengers moving from Manhattan to JFK. By the time our in-flight survey was conducted in 1963, Carey's proportion had declined to 46%. We estimate Carey's percentage of the Manhattan market to have further declined to 28% in 1967. This, of course, excludes all transfer passengers. It is, therefore, a very significant decline in mass transportation usage.

2. Taxi use to LaGuardia

As we suspected, the use of taxis between Manhattan and LaGuardia Airport is somewhat higher than the 45% figure I indicated to JFK. In 1967, approximately 67% of air passengers going from Manhattan to LaGuardia arrived at the Airport via taxis. This was a small growth from the 65% in 1963 and a growth of 10 percentage points from the 57% in 1956.

This review of access systems for major world airports at present and in the immediate future has re-emphasized the overwhelming role of highway transport and the very minor role of existing alternatives. Such things as the decrease in the New York area of airport bus passengers, despite major highway congestion, may emphasize the absence of attractive alternatives other than the automobile and taxi, but the extent to which the most convenient high-speed, nonstop airport CBD service will attract the passengers away from "their own wheels" must certainly be evaluated with extreme caution. The economics of such alternatives also demand careful attention since there may be the need to justify very heavy subsidy for construction and operation on the basis of some sort of total system or total transportation concept. Finally, there is every indication that airport employees and their private vehicles will become an increasing and well-nigh irreducible portion of the airport traffic and parking problems of the future.

Criterion for Tolerable Roughness in a Laminar Boundary Layer

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Nomenclature

k = height of roughness element
R_{xk} $\equiv x_k U/\nu$
R_{xNT} $\equiv x_{NT}U/\nu$
R_k $\equiv kU/\nu$
u_k = velocity at the height k and at the roughness position in the absence of the roughness
U = stream velocity
x_k = distance from the starting point of the boundary layer to the roughness position
x_{NT} = distance from the starting point of the boundary layer to the position of natural transition
ν = kinematic coefficient of viscosity

1. Introduction

There has long been interest in the height of surface roughness that an incompressible laminar boundary layer can tolerate without transition being affected. A satisfactory measure has been sought and, as an example, Head[1] has advocated, as more generally useful, a criterion based upon a value of $u_k k/\nu$ in preference to one based upon a value of Uk/ν. It is important in this context first to define what is meant by "tolerable" roughness and second, to distinguish between the effects of two-dimensional and three-dimensional roughness shapes.

It has been pointed out[2,3] that the general problem of the effect of roughness upon transition is expressible in terms of three nondimensional groups. The case of the tolerable roughness is associated with a movement of transition forward from its naturally occuring position; that is, it is associated with a fixed value of one of the three nondimensional

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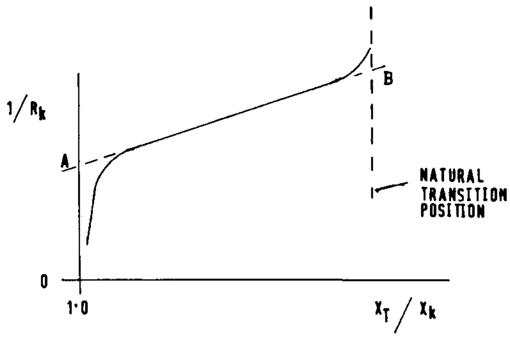

Fig. 1 Sketch to illustrate the definition of two reference Reynolds numbers.

groups—say the value of the Reynolds number at natural transition R_{xNT}. Thus, it is expressible in terms of two nondimensional groups.

2. Two-Dimensional Roughness

Figure 1 is a sketch indicating an example of the results for two-dimensional wire roughnesses that are discussed in Ref. 3. It is a plot of the wire Reynolds number, $R_k \equiv Uk/\nu$, against the position of the beginning of transition, x_T. As the value of R_k rises, transition gradually moves forward to the wire position. An effective value of R_k for transition at the wire can be defined as the value at point A in this figure. As shown in Ref. 3, the corresponding value of R_k is a constant for low turbulence flow over a flat plate, with a zero pressure gradient.

As the value of R_k falls, the position of transition moves rearwards towards the position of naturally occurring transition. A definition of the tolerable roughness can be given as the value of R_k corresponding to point B. It is seen that, because the position of transition moves forward steadily from the naturally occurring position as the roughness size increases from zero, a definition of a finite tolerable roughness must be arbitrary. In contrast to the case of transition at the wire, this tolerable roughness Reynolds number is not a constant, but becomes a function of the second nondimensional group. This is illustrated in Fig. 2, where these results for two-dimensional wire roughnesses are taken from Ref. 3. It is seen that R_k is a function of the Reynolds number of the wire position as indicated by R_{xk}. Nor is a single value of $u_k k/\nu$ obtained for the tolerable roughness condition. This is illustrated in Fig. 3, where the curve of Fig. 2. is replotted, showing that similarly $u_k k/\nu$ is a function of R_{xk}.

3. Three-Dimensional Roughness

A similar definition of a tolerable roughness Reynolds number can be made for three-dimensional roughness shapes. Figure 4 is a plot of some of the results of Tani, Komoda, and

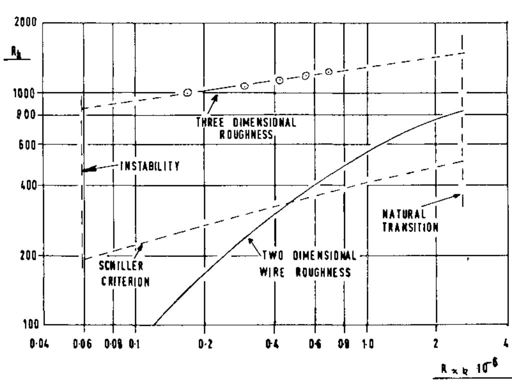

Fig. 2 Variation of roughness Reynolds number for two- and three-dimensional shapes.

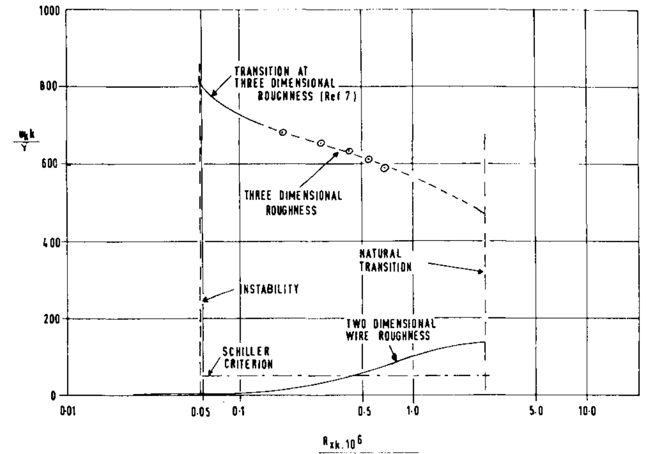

Fig. 3 Variation of roughness Reynolds number for two- and three-dimensional shapes.

Iuchi[4] for positions of a midpoint in the transition region. Again, a linear plot is obtained, and so the tolerable roughness can be indicated by the value of the Reynolds number corresponding to point B.

It is found also for the three-dimensional shape that neither a constant value of R_k nor a constant value of $u_k k/\nu$ is obtained as a criterion for the tolerable roughness. This can be seen in Figs. 2 and 3, where the values obtained from these results of Tani, Komoda, and Iuchi are presented. Both types of roughness Reynolds number are seen to be functions of R_{xk}.

4. Schiller's Criterion

The much-used criterion for tolerable roughness suggested by Schiller, that is, $u_k k/\nu$ = const, has been discussed for two-dimensional wire roughness shapes in Ref. 3, where it was shown to be unsatisfactory. Its value of $u_k k/\nu = 50$, suggested by Gazeley[6] is plotted in Fig. 2, where it is seen to be too drastic a criterion for three-dimensional shapes, and to disagree with the results for two-dimensional shapes by lying partly above and partly below the criterion given.

5. The Tolerable Roughness Criterion

It is now seen that even with a low turbulence stream and a zero pressure gradient, there is not a simple criterion for tolerable roughness, in the form either of a unique value of $u_k k/\nu$, or of a unique value of R_k. The use of constant values for these two Reynolds numbers provides at the best only a

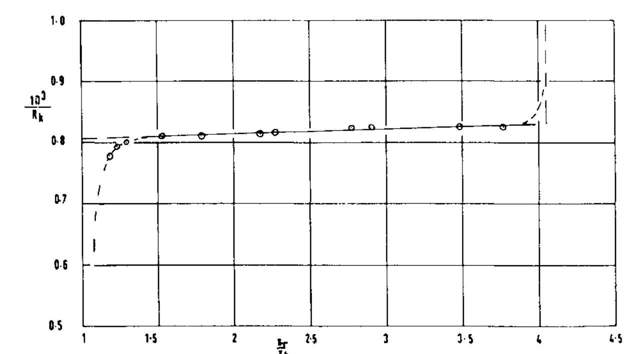

Fig. 4 Movement of transition position with variation of roughness Reynolds number of a three-dimensional shape. (Results of Ref. 4. Cylindrical shape x_k = 56 cm; k = 0.1 cm.)

† The dotted extension to the right is added in this figure to indicate a typical trend of experimental results such as were obtained by Klebanoff, Schubauer, and Tidstrom.[5]

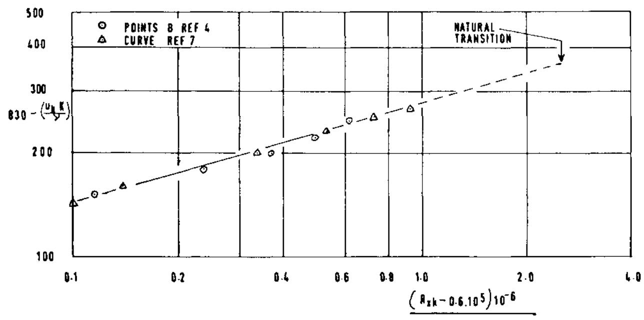

Fig. 5 Correlation for roughness Reynolds number of three-dimensional shapes.

rough guide to the size of the tolerable roughness. Accurate estimates require the specification of another nondimensional group.

For a two-dimensional roughness, the criterion given in Ref. 3 and illustrated in Figs. 2 and 3 is given by

$$826/R_k = 1 + \tfrac{1}{3}\{(2.6 \cdot 10^6/R_{xk}) - 1\}$$

For a three-dimensional roughness, the experimental results of Tani et al., that are shown in Fig. 3 can be compared with the smooth curve drawn by Tani[7] through the experimental results for the condition when transition is at the roughness element, that is, the values corresponding to point A in Fig. 4. The difference is small because, with an isolated roughness element, transition moves forward very rapidly towards the element. There is evidence that $u_k k/\nu$ tends to a value of about 830 as R_{xk} approaches the value of $0.6 \cdot 10^5$ that corresponds to the lower stability limit of a laminar boundary layer.[8] A suitable correlation is illustrated in Fig. 5 where the straight line is given by

$$830 - (u_k k/\nu) = 275[(R_{xk} - 0.6 \cdot 10^5)10^{-6}]^{0.29} \quad (1)$$

If this relation is valid beyond the range of the experimental results used, then with the roughness at the position of natural transition where $R_{xNT} = 2.6 \cdot 10^6$, the corresponding value of $u_k k/\nu$ tends to 470.

The same results plotted in Fig. 2 fit the relation,

$$R_k = 1300\,(R_{xk} \cdot 10^{-6})^{0.15} \quad (2)$$

This equation is simpler than Eq. (1), it is more convenient for design studies, as R_k is based upon the stream velocity, and it agrees with Eq. (1) to within about ±2% over the whole range from the instability position to the natural transition position. At the former position it gives an R_k of 850, and at the latter one, of 1500.

6. Comparison of Two-and Three-Dimensional Shapes

There is a difference in the effects of two- and three-dimensional roughness shapes that is of practical interest when a roughness element is close to the starting point of the boundary layer. The present evidence suggests that a two-dimensional cylindrical element can bring transition forward to itself even when the element is at the leading edge, for it is able, by virtue of its drag, to add the necessary ΔR_θ to the boundary layer as well as distorting the boundary-layer profile towards instability conditions. By contrast, it seems that an isolated roughness has a negligible effect upon the momentum thickness of the boundary layer and so, as noted in Ref. 5, it appears to become ineffective when R_{xk} is at or below the value corresponding to the lower limit of stability of the laminar boundary layer.

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Thin Airfoil in Nonuniform Parallel Streams

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1. Introduction

The small disturbance theory for two parallel streams was initiated by von Kármán.[1] The undisturbed streams with velocity U_1 and U_2 are separated by the streamline represented by the x axis. A thin airfoil represented by a single vortex of strength Γ was located as in the lower stream with velocity U_1 at a distance d below the origin. According to the small disturbance theory, the condition of matching pressure $U_1u'_1 = U_2u'_2$ and that of matching slope $v'_1/U_1 = v'_2/U_2$ are imposed along the undisturbed streamline, the x axis. These boundary conditions are fulfilled by the introduction of images.[1] The disturbance velocity potential for the lower stream is

$$w_1(z) = \Gamma/(2\pi i)\ln(z + di) + \lambda_1\Gamma/(2\pi i)\ln(z - di) \quad (1)$$

where $z = x + iy$ and $\lambda_1 = (U_1^2 - U_2^2)/(U_1^2 + U_2^2)$. The second term represented the reflected image located at $z = di$ with strength $\lambda_1\Gamma$. The disturbance velocity potential for the upper stream is

$$w_2(z) = \Gamma\lambda_2/(2\pi i)\ln(z + di) \quad (2)$$

It represents the "diffracted" disturbance with $\lambda_2 = 2U_1U_2/(U_1^2 + U_2^2)$. It should be noted that $|\lambda_1|$ is less than unity and is equal to unity only in the two limiting cases: 1) $U_2/U_1 = 0$, $\lambda_1 = 1$, and line $y = 0$ is a constant pressure free streamline and 2) $U_2/U_1 \to \infty$, $\lambda_1 = -1$, and line $y = 0$ represents a solid wall. Extensions of the method of images to a flowfield of three parallel streams, i.e., two dividing streamlines as shown in Fig. 1, have been made[2] for the special case of jets ($U_1 = U_3 = 0$) and wakes ($U_1 = U_3 \neq U^2$). In this Note, a system of images will be formulated for the general case $U_1 \neq U_2 \neq U_3$ and the airfoil will be represented by a vorticity distribution. The distribution will be determined in a similar manner for an airfoil in a uniform

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